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A note on isomorphism rigidity for Banach algebras defined by multipliers

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ABSTRACT

Let $\mathcal{A}$ be a Banach algebra and T a bounded two-sided multiplier on $\mathcal{A}$. We study the Banach algebra $\mathcal{A}_T$ with product $a \star_T b = aT(b)$ and ask what follows from the existence of a Banach algebra isomorphism $\mathcal{A}_T \cong \mathcal{A}$. For a right faithful Banach algebra, such an isomorphism forces T to be injective. If, moreover, $\overline{\operatorname{span}(\mathcal{A}^2)} = \mathcal{A}$, it also forces $\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}$. In the unital case, these necessary conditions become a complete characterization: $\mathcal{A}_T \cong \mathcal{A}$ if and only if T is invertible in the multiplier algebra. Equivalently, if $T(a) = ma$ with $m = T(1)$, then $\mathcal{A}_T \cong \mathcal{A}$ iff m is invertible in $\mathcal{A}$. We also derive corresponding consequences for semigroup algebras. For a discrete semigroup S and $t \in S$, the isomorphism $\ell^1(S)_{L_t} \cong \ell^1(S)$ forces injectivity of the left translation $s \mapsto ts$ whenever $\ell^1(S)$ is right faithful and, under the same multiplier hypotheses, yields a natural obstruction in terms of the ideal StS.

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1. Introduction

The study of Banach algebras obtained from a multiplier by modifying the multiplication has a background in the general theory of multipliers; see, for example, Larsen [8, 9]. An explicit study of Banach algebras of the form $\mathcal{A}_T$, with product $a \star_T b = aT(b)$, was carried out by Laali [7], who investigated their Arens regularity and amenability properties. Subsequent work, including that of Ebadian, Jabbari and Shams [5], studied further analytic and homological properties of $\mathcal{A}_T$, including approximate identities, amenability-related properties, and related homological questions. In these works one starts with a prescribed multiplier T and studies the resulting algebra $\mathcal{A}_T$.

In this note we consider the converse direction. Given a Banach algebra $\mathcal{A}$ and a bounded two-sided multiplier T on $\mathcal{A}$, one defines a new product on the underlying Banach space of $\mathcal{A}$ by

$$a \star_T b = aT(b) \qquad (a, b \in \mathcal{A}).$$

The multiplier identities $T(ab) = aT(b) = T(a)b$ guarantee associativity, and the resulting Banach algebra is denoted $\mathcal{A}_T$. Instead of asking which properties of $\mathcal{A}$ pass to $\mathcal{A}_T$, we ask what the existence of an isomorphism $\mathcal{A}_T \cong \mathcal{A}$ reveals about the defining multiplier T.

Our aim is to isolate some elementary rigidity consequences of this isomorphism hypothesis. In the non-unital case, under natural assumptions, the existence of an isomorphism $\mathcal{A}_T \cong \mathcal{A}$ forces T to be injective and imposes a density condition on the ideal $\mathcal{A}T(\mathcal{A})$. These conditions are not sufficient in general, as a simple example based on c_0 shows. The principal result is Theorem 2.7: in the unital case, the isomorphism $\mathcal{A}_T \cong \mathcal{A}$ holds if and only if T is invertible in the multiplier algebra, equivalently if and only if $T(1)$ is invertible in $\mathcal{A}$.

The final section applies these abstract observations to semigroup algebras $\ell^1(S)$. For a discrete semigroup S and $t \in S$, the left translation operator $L_t(f) = \delta_t * f$ defines such a deformation, and the general obstructions translate into restrictions on the map $s \mapsto ts$. In the corresponding two-sided multiplier setting, they also give an obstruction expressed in terms of the principal ideal StS. These applications are included as illustrations of the general theory, not as a separate classification program.

General background on multipliers and Banach algebras can be found in [1, 3, 10].

2. General Banach Algebra Results

Throughout, $\mathcal{A}$ denotes a Banach algebra, and $\mathcal{M}(\mathcal{A})$ denotes the algebra of bounded two-sided multipliers on $\mathcal{A}$. So $T \in \mathcal{M}(\mathcal{A})$ means

$$T(ab) = aT(b) = T(a)b \qquad (a, b \in \mathcal{A}).$$

The right annihilator of $\mathcal{A}$ is

$$\operatorname{ann}_r(\mathcal{A}) = \{a \in \mathcal{A} : \mathcal{A}a = \{0\}\}.$$

We say $\mathcal{A}$ is *right faithful* if $\operatorname{ann}_r(\mathcal{A}) = \{0\}$. Also $\mathcal{A}$ is *generated by products in norm* if $\overline{\operatorname{span}(\mathcal{A}^2)} = \mathcal{A}$.

For $T \in \mathcal{M}(\mathcal{A})$, the formula

$$a \star_T b = aT(b) \qquad (a, b \in \mathcal{A})$$

defines a Banach algebra structure on the underlying Banach space of $\mathcal{A}$, which we denote by $\mathcal{A}_T$. Since each product in $\mathcal{A}_T$ has the form $aT(b)$, one has

$$\operatorname{span}(\mathcal{A}_T^2) \subseteq \mathcal{A}T(\mathcal{A}).$$

If $\mathcal{A}$ is unital, then every two-sided multiplier is implemented by a central element of $\mathcal{A}$. Indeed, if 1 is the identity of $\mathcal{A}$ and $m = T(1)$, then

$$T(a) = T(1a) = T(1)a = ma \quad \text{and} \quad T(a) = T(a1) = aT(1) = am$$

for all $a \in \mathcal{A}$. So $m \in Z(\mathcal{A})$ and $T(a) = ma = am$ for every $a \in \mathcal{A}$.

We begin with two general necessary conditions for the existence of an isomorphism $\mathcal{A}_T \cong \mathcal{A}$.

Lemma 2.1. *Let $\mathcal{A}$ be a Banach algebra and $T \in \mathcal{M}(\mathcal{A})$. Then*

$$\operatorname{Ker}(T) \subseteq \operatorname{ann}_r(\mathcal{A}_T).$$

Proof. If $a \in \operatorname{Ker}(T)$, then $T(a) = 0$. Hence, for every $b \in \mathcal{A}$,

$$b \star_T a = bT(a) = 0.$$

Thus $a \in \operatorname{ann}_r(\mathcal{A}_T)$. $\square$

Proposition 2.2. *Let $\mathcal{A}$ be a right faithful Banach algebra. If $\mathcal{A}_T \cong \mathcal{A}$ as Banach algebras, then T is injective.*

Proof. Let $\Phi : \mathcal{A}_T \to \mathcal{A}$ be a Banach algebra isomorphism. Since algebra isomorphisms preserve right annihilators,

$$\Phi(\operatorname{ann}_r(\mathcal{A}_T)) = \operatorname{ann}_r(\mathcal{A}).$$

Indeed, if $x \in \operatorname{ann}_r(\mathcal{A}_T)$ and $a \in \mathcal{A}$, choose $b \in \mathcal{A}_T$ such that $\Phi(b) = a$. Then

$$a\Phi(x) = \Phi(b)\Phi(x) = \Phi(b \star_T x) = 0,$$

so $\Phi(x) \in \operatorname{ann}_r(\mathcal{A})$. The reverse inclusion follows by applying the same argument to Φ^{-1}.

Since $\mathcal{A}$ is right faithful, $\operatorname{ann}_r(\mathcal{A}) = \{0\}$. Hence $\operatorname{ann}_r(\mathcal{A}_T) = \{0\}$, and Lemma 2.1 gives $\operatorname{Ker}(T) = \{0\}$. $\square$

So injectivity of T is necessary whenever $\mathcal{A}$ is right faithful. It is not sufficient in general; see Example 4.3.

A second obstruction comes from the closed span of products.

Lemma 2.3. *Let $\mathcal{A}$ be a Banach algebra and $T \in \mathcal{M}(\mathcal{A})$. Then*

$$\overline{\operatorname{span}(\mathcal{A}_T^2)} \subseteq \overline{\mathcal{A}T(\mathcal{A})}.$$

Proof. Each product in $\mathcal{A}_T$ has the form $a \star_T b = aT(b)$, which lies in $\mathcal{A}T(\mathcal{A})$. Taking linear spans and closures gives the result. $\square$

Proposition 2.4. *Let $\mathcal{A}$ be a Banach algebra such that $\overline{\operatorname{span}(\mathcal{A}^2)} = \mathcal{A}$. If $\mathcal{A}_T \cong \mathcal{A}$ as Banach algebras, then*

$$\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}.$$

Proof. Let $\Phi : \mathcal{A}_T \to \mathcal{A}$ be a Banach algebra isomorphism. Since Φ preserves products, it maps $\operatorname{span}(\mathcal{A}_T^2)$ onto $\operatorname{span}(\mathcal{A}^2)$. By continuity of Φ and Φ^{-1},

$$\Phi\big(\overline{\operatorname{span}(\mathcal{A}_T^2)}\big) = \overline{\operatorname{span}(\mathcal{A}^2)} = \mathcal{A}.$$

Hence $\overline{\operatorname{span}(\mathcal{A}_T^2)} = \mathcal{A}$. By Lemma 2.3,

$$\mathcal{A} = \overline{\operatorname{span}(\mathcal{A}_T^2)} \subseteq \overline{\mathcal{A}T(\mathcal{A})} \subseteq \mathcal{A},$$

and therefore $\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}$. $\square$

The two propositions above give only necessary conditions. They are not sufficient in general.

Remark 2.5. *There exist a commutative non-unital Banach algebra $\mathcal{A}$ and an injective multiplier $T \in \mathcal{M}(\mathcal{A})$ such that $\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}$ but $\mathcal{A}_T \ncong \mathcal{A}$; see Example 4.3.*

Remark 2.6. *The hypotheses in Propositions 2.2 and 2.4 cannot simply be omitted. Example 4.1 shows that, without right faithfulness, an isomorphism $\mathcal{A}_T \cong \mathcal{A}$ need not force T to be injective. The same example also shows that, without the assumption $\overline{\operatorname{span}(\mathcal{A}^2)} = \mathcal{A}$, the density conclusion $\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}$ need not follow from $\mathcal{A}_T \cong \mathcal{A}$.*

In the unital case, the situation is completely explicit.

Theorem 2.7. *Let $\mathcal{A}$ be a unital Banach algebra and $T \in \mathcal{M}(\mathcal{A})$. Write*

$$T(a) = ma = am \qquad (a \in \mathcal{A}),$$

where $m = T(1) \in Z(\mathcal{A})$. Then $\mathcal{A}_T \cong \mathcal{A}$ as Banach algebras iff m is invertible in $\mathcal{A}$. Equivalently, $\mathcal{A}_T \cong \mathcal{A}$ iff T is invertible in $\mathcal{M}(\mathcal{A})$.

Proof. Suppose first that m is invertible in $\mathcal{A}$. Define $\Psi : \mathcal{A}_T \to \mathcal{A}$ by

$$\Psi(a) = ma \qquad (a \in \mathcal{A}).$$

Then Ψ is a bounded bijective linear map. For $a, b \in \mathcal{A}$,

$$\Psi(a \star_T b) = \Psi(aT(b)) = \Psi(amb) = mamb,$$

while

$$\Psi(a)\Psi(b) = (ma)(mb) = mamb,$$

since m is central. Thus Ψ is a Banach algebra isomorphism.

Conversely, suppose $\Phi : \mathcal{A}_T \to \mathcal{A}$ is a Banach algebra isomorphism. Since $\mathcal{A}$ is unital, $\mathcal{A}_T$ is also unital. Let $u \in \mathcal{A}$ be the identity of $\mathcal{A}_T$. Then, for every $a \in \mathcal{A}$,

$$u \star_T a = a \quad \text{and} \quad a \star_T u = a.$$

Using $T(a) = ma = am$, these become

$$uma = a \quad \text{and} \quad amu = a \qquad (a \in \mathcal{A}).$$

Taking $a = 1$ gives

$$um = 1 \quad \text{and} \quad mu = 1.$$

Hence m is invertible in $\mathcal{A}$.

Finally, in the unital case, T is invertible in $\mathcal{M}(\mathcal{A})$ iff the implementing element $m = T(1)$ is invertible in $\mathcal{A}$. □

Remark 2.8. *Theorem 2.7 subsumes the two general obstructions in the unital setting. If m is invertible, then $T(a) = ma$ is injective and*

$$\mathcal{A}T(\mathcal{A}) = \mathcal{A}m\mathcal{A} = \mathcal{A}.$$

Thus Propositions 2.2 and 2.4 are mainly relevant in the non-unital case.

3. Semigroup Applications

Let S be a discrete semigroup, and let $\ell^1(S)$ denote its semigroup algebra with convolution product. For $t \in S$, define

$$\lambda_t : S \to S, \qquad \lambda_t(s) = ts,$$

and

$$L_t : \ell^1(S) \to \ell^1(S), \qquad L_t(f) = \delta_t * f.$$

The corresponding deformed product on $\ell^1(S)$ is

$$f \star_t g = f * (\delta_t * g).$$

The results of Section 2 apply whenever L_t is a two-sided multiplier. We use two standard facts: $\ell^1(S)$ is right faithful iff S is right reductive, and $\overline{\mathrm{span}(\ell^1(S)^2)} = \ell^1(S)$ iff $S^2 = S$; see [2–4, 6].

Proposition 3.1. *Let S be a discrete semigroup and $t \in S$. Then $L_t : \ell^1(S) \to \ell^1(S)$ is injective iff $\lambda_t : S \to S$ is injective.*

Proof. If λ_t is not injective, choose $s_1 \neq s_2$ with $ts_1 = ts_2$. Then

$$L_t(\delta_{s_1} - \delta_{s_2}) = \delta_{ts_1} - \delta_{ts_2} = 0,$$

so L_t is not injective.

Conversely, suppose λ_t is injective and let $f \in \ell^1(S)$ satisfy $L_t(f) = 0$. Write

$$f = \sum_{s \in S} f(s)\delta_s.$$

Then

$$0 = L_t(f) = \sum_{s \in S} f(s)\delta_{ts}.$$

Since the elements ts are pairwise distinct, all coefficients $f(s)$ must vanish. Thus $f = 0$. □

Proposition 3.2. *Let S be a right reductive discrete semigroup with $S^2 = S$, and let $t \in S$. Assume L_t is a two-sided multiplier. If*

$$\ell^1(S)_{L_t} \cong \ell^1(S),$$

then λ_t is injective and $StS = S$.

Proof. Since S is right reductive, $\ell^1(S)$ is right faithful. Hence Proposition 2.2 gives that L_t is injective. By Proposition 3.1, λ_t is injective.

Since $S^2 = S$, we have $\overline{\mathrm{span}(\ell^1(S)^2)} = \ell^1(S)$. Proposition 2.4 therefore gives

$$\overline{\ell^1(S)L_t(\ell^1(S))} = \ell^1(S).$$

Now $L_t(\ell^1(S)) = \ell^1(tS)$, because $L_t(\delta_s) = \delta_{ts}$ for each $s \in S$. It follows that

$$\ell^1(S)L_t(\ell^1(S)) = \mathrm{span}\{\delta_x * \delta_{ty} : x, y \in S\} = \mathrm{span}\{\delta_{xty} : x, y \in S\}.$$

Hence its closed linear span is $\ell^1(StS)$. So

$$\ell^1(StS) = \ell^1(S),$$

which is equivalent to $StS = S$. □

In the monoid case one can argue directly, without any two-sided multiplier assumption.

Proposition 3.3. *Let S be a monoid with identity e, and $t \in S$. The following are equivalent:*

(1) $\ell^1(S)_{L_t} \cong \ell^1(S)$ *as Banach algebras;*
(2) δ_t *is invertible in* $\ell^1(S)$*;*
(3) t *is a unit of* S.

Proof. First assume δ_t is invertible, and let $\eta \in \ell^1(S)$ satisfy

$$\eta * \delta_t = \delta_e = \delta_t * \eta.$$

Define

$$\Psi(f) = f * \delta_t \qquad (f \in \ell^1(S)).$$

Then Ψ is a bounded bijective linear map with inverse $\Psi^{-1}(h) = h * \eta$. For $f, g \in \ell^1(S)$,

$$\Psi(f \star_t g) = \Psi(f * (\delta_t * g)) = (f * (\delta_t * g)) * \delta_t,$$

while

$$\Psi(f)\Psi(g) = (f * \delta_t) * (g * \delta_t).$$

By associativity of convolution,

$$(f * (\delta_t * g)) * \delta_t = f * ((\delta_t * g) * \delta_t) = f * (\delta_t * (g * \delta_t)) = (f * \delta_t) * (g * \delta_t).$$

Thus Ψ is an algebra isomorphism.

Conversely, suppose

$$\Phi : \ell^1(S)_{L_t} \to \ell^1(S)$$

is a Banach algebra isomorphism. Since $\ell^1(S)$ is unital, so is $\ell^1(S)_{L_t}$. Let u be the identity of $\ell^1(S)_{L_t}$. Then

$$u \star_t \delta_e = \delta_e \qquad \text{and} \qquad \delta_e \star_t u = \delta_e.$$

Since

$$u \star_t \delta_e = u * (\delta_t * \delta_e) = u * \delta_t$$

and

$$\delta_e \star_t u = \delta_e * (\delta_t * u) = \delta_t * u,$$

we get

$$u * \delta_t = \delta_e \qquad \text{and} \qquad \delta_t * u = \delta_e.$$

Hence δ_t is invertible in $\ell^1(S)$.

Finally, δ_t is invertible in $\ell^1(S)$ iff t is a unit of S. Indeed, if $\delta_t * f = \delta_e = f * \delta_t$, then $e \in tS$ and $e \in St$, so there exist $s, r \in S$ with

$$ts = e, \qquad rt = e.$$

Hence

$$r = r(ts) = (rt)s = s,$$

so $ts = st = e$. Thus t is a unit. The converse is clear. ☐

4. Examples

We end with three elementary examples showing the role and the limitations of the preceding obstructions.

Example 4.1. *Let $\mathcal{A}$ be a Banach space with zero product. Then $\operatorname{ann}_r(\mathcal{A}) = \mathcal{A}$, so $\mathcal{A}$ is not right faithful. Every bounded linear operator on $\mathcal{A}$ is a two-sided multiplier. If T is a non-injective bounded projection, then*

$$a \star_T b = aT(b) = 0 \qquad (a, b \in \mathcal{A}).$$

Thus $\mathcal{A}_T = \mathcal{A}$ as Banach algebras, although T is not injective.

Example 4.2. *Let $\mathcal{A} = C([0,1])$ with pointwise multiplication, and define*

$$T(f)(x) = xf(x).$$

Then T is an injective multiplier. But

$$\mathcal{A}T(\mathcal{A}) = \{f \in C([0,1]) : f(0) = 0\},$$

which is a proper closed ideal of $\mathcal{A}$.

If $\mathcal{A}_T$ were isomorphic to $\mathcal{A}$, then $\mathcal{A}_T$ would be unital. Let u be its identity. From $u \star_T 1 = 1$ we would have

$$u(x)x = 1 \qquad (x \in [0,1]),$$

impossible for a continuous function u. Hence $\mathcal{A}_T \ncong \mathcal{A}$.

Example 4.3. *Let $\mathcal{A} = c_0(\mathbb{N})$ with pointwise multiplication, and define*

$$T(f)(n) = \frac{1}{n} f(n) \qquad (n \in \mathbb{N}).$$

Then T is a multiplier, implemented by the bounded sequence

$$w = \left(\frac{1}{n}\right)_{n \geq 1} \in \ell^\infty.$$

Since $w(n) \neq 0$ for all n, the operator T is injective.

We claim $\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}$. Let $h \in c_{00}$. Define

$$f = \mathbf{1}_{\operatorname{supp}(h)} \in c_{00}, \qquad g(n) = nh(n).$$

Since h has finite support, $g \in c_{00} \subseteq c_0$. Moreover, $T(g) = h$, so $fT(g) = h$. Hence $c_{00} \subseteq \mathcal{A}T(\mathcal{A})$. As c_{00} is dense in c_0, it follows that

$$\overline{\mathcal{A}T(\mathcal{A})} = \mathcal{A}.$$

Nevertheless, $\mathcal{A}_T \ncong \mathcal{A}$. The algebra c_0 has a bounded approximate identity, but $\mathcal{A}_T$ has no bounded left approximate identity. Indeed, suppose (u_α) is a bounded left approximate identity for $\mathcal{A}_T$, with

$$\|u_\alpha\|_\infty \leq C$$

for some $C > 0$. Let e_n be the nth coordinate vector. Since

$$u_\alpha \star_T e_n \to e_n,$$

we have

$$\frac{u_\alpha(n)}{n} \to 1 \qquad \text{for each fixed } n.$$

Hence, for each n, there exists α_n such that

$$|u_\alpha(n)| > \frac{n}{2} \qquad (\alpha \geq \alpha_n).$$

Choosing $n > 2C$ *gives a contradiction. So* $\mathcal{A}_T$ *has no bounded left approximate identity.*

If $\Phi : \mathcal{A} \to \mathcal{B}$ *is a Banach algebra isomorphism and* (e_α) *is a bounded left approximate identity for* $\mathcal{A}$*, then* $(\Phi(e_\alpha))$ *is a bounded left approximate identity for* $\mathcal{B}$*. Thus the existence of a bounded left approximate identity is preserved under Banach algebra isomorphisms. Since* c_0 *has one and* $\mathcal{A}_T$ *does not, we conclude*

$$\mathcal{A}_T \ncong \mathcal{A}.$$

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