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Existence and uniqueness of approximate quartic reciprocal mappings

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ABSTRACT

We present a novel form of quartic reciprocal functional equations in this work. The intriguing fact that harmonic points arise in projective geometry inspired our work to examine this new equation. The stability of this equation is established using the fixed point method. Additionally, several upper bounds are considered to produce stability results. As an application of this equation, it is connected to the Rayleigh scattering formula. The arguments of this equation are connected to harmonic points at the conclusion of this research.

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1. Introduction

The stability problem of functional equations was first raised by Ulam [14] in 1940 at the University of Wisconsin. He asked whether an approximate solution of a functional equation can be close to an exact solution. The following year, Hyers [6] gave the most powerful and affirmative response to Ulam's query about Banach spaces. In 1978, Aoki [1] expanded Hyers' finding to include bounded functions. Hyers' theorem was expanded by Rassias [9] by taking into account an unbounded control function; see also [5].

There have been notable advancements in the study of the stability of reciprocal functional equations in various Banach spaces over the years. In other words, Ravi and the second author [10] defined the reciprocal functional equation for the first time as follows:

$$R(r+s)=\frac{R(r)R(s)}{R(r)+R(s)}. \tag{1.1}$$

They obtained some stability results of a mapping on the space of non-zero real numbers. It is obvious that the mapping $R:\mathbb{R}\backslash\{0\}\longrightarrow\mathbb{R}$ defined via $R(r)=\frac{c}{r}$ is a solution of (1.1). Next, Jung [7] established the stability of (1.1) by using a fixed point technique.

The first author and Kim [3] introduced the quadratic reciprocal functional equation

$$\mathcal{Q}(2u+v)+\mathcal{Q}(2u-v)=\frac{2\mathcal{Q}(u)\mathcal{Q}(v)[4\mathcal{Q}(v)+\mathcal{Q}(u)]}{(4\mathcal{Q}(v)-\mathcal{Q}(u))^2} \tag{1.2}$$

for the first time. They also studied the Găvruta and Rassias stabilities for (1.2) on non-zero real numbers; for modified version of (1.2), we refer to [2]. Next, a reciprocal-cubic functional equation of the form

$$C(2\vartheta_1+\vartheta_2)+C(\vartheta_1+2\vartheta_2)=\frac{9C(\vartheta_1)C(\vartheta_2)\left[C(\vartheta_1)+C(\vartheta_2)+2C(\vartheta_1)^{\frac{1}{3}}C(\vartheta_2)^{\frac{1}{3}}\left(C(\vartheta_1)^{\frac{1}{3}}+C(\vartheta_2)^{\frac{1}{3}}\right)\right]}{\left[2C(\vartheta_1)^{\frac{2}{3}}+2C(\vartheta_2)^{\frac{2}{3}}+5C(\vartheta_1)^{\frac{1}{3}}C(\vartheta_2)^{\frac{1}{3}}\right]^3} \tag{1.3}$$

was dealt with in [8] and also a general form of reciprocal-quartic was recently presented in [11]. Note that other functional equation associated to reciprocal-cubic mappings as solutions and moreover fuzzy approximations of a multiplicative inverse cubic functional equations were considered in [13].

Recall from [8] that the first model of reciprocal-quartic functional equation is given as

$$Q(2r+s)+Q(2r-s)=\frac{2Q(r)Q(s)\left[Q(r)+16Q(s)+24\sqrt{Q(r)Q(s)}\right]}{\left[4\sqrt{Q(s)}-\sqrt{Q(r)}\right]^4}. \tag{1.4}$$

It is easily verified that the reciprocal-cubic function $C(r)=\frac{1}{r^3}$ and the reciprocal-quartic function $Q(r)=\frac{1}{r^4}$ are solutions of functional equations (1.3) and (1.4), respectively.

In this paper, we consider the rational form of quartic reciprocal functional equation

$$Q\left(\frac{pq}{p+2q}\right)+Q\left(\frac{pq}{p-2q}\right)=Q\left(\frac{pq}{2p+q}\right)+Q\left(\frac{pq}{2p-q}\right)+30Q(p)-30Q(q). \tag{1.5}$$

We study the general solutions of (1.5) and in continuation, we establish its various stabilities. As an application of this equation, it is connected to the Rayleigh scattering formula. The arguments of this equation are connected to harmonic points at the conclusion of this research.

2. Fundamental results and stability with a fixed point approach

Here and subsequently, we assume that

$$\mathbb{A} = \{p \in \mathbb{R} - \{0\} : p + 2q \neq 0, p - 2q \neq 0, 2p + q \neq 0, 2p - q \neq 0\},$$

for all $p, q \in \mathbb{R}$. In order to arrive at a significant fundamental property of equation (1.5), let us rewrite it as follows:

$$Q\left(\frac{q}{1+\frac{2q}{p}}\right) + Q\left(\frac{q}{1-\frac{2q}{p}}\right) = Q\left(\frac{q}{2+\frac{q}{p}}\right) + Q\left(\frac{q}{2-\frac{q}{p}}\right) + 30Q(p) - 30Q(q). \tag{2.1}$$

Further in the following results, we assume that $Q : \mathbb{A} \longrightarrow \mathbb{R}$ is a mapping with the condition that $Q(p) \to 0$ as $p \to \infty$.

Proposition 2.1. *Suppose that a mapping Q satisfies (2.1). Then, for any positive integer m, the mapping Q also satisfies the general functional equations $Q\left(\frac{p}{2^m}\right) = 2^{4m}Q(p)$, for all $p \in \mathbb{A}$.*

Proof. By our assumption that as $p \to \infty$ and then replacing q with p, it is easy to arrive at $Q\left(\frac{p}{2}\right) = 16Q(p)$, for all $p \in \mathbb{A}$. One can easily prove that using induction process for any positive integer m that $Q\left(\frac{p}{2^m}\right) = 2^{4m}Q(p)$, for all $p \in \mathbb{A}$. □

One can check that the mapping $Q : \mathbb{A} \longrightarrow \mathbb{R}$ defined through $Q(x) = \frac{c}{x^4}$ is a solution of (1.5).

Definition 2.2. *A mapping Q is called as a quartic reciprocal mapping if it satisfies equation (1.5). Also, we call equation (1.5) to be a quartic reciprocal (rational form) functional equation.*

Let Ω be a set. A function $d : \Omega \times \Omega \longrightarrow [0, \infty]$ is said to be a *generalized metric* on Ω if and only if d satisfies the statements

(i) $d(\eta, \vartheta) = 0$ if and only if $\eta = \vartheta$ for all $\eta, \vartheta \in \Omega$;
(ii) $d(\eta, \vartheta) = d(\vartheta, \eta)$ for all $\eta, \vartheta \in \Omega$;
(iii) $d(\eta, \xi) \leq d(\eta, \vartheta) + d(\vartheta, \xi)$ for all $\eta, \vartheta, \xi \in \Omega$.

Note that the only substantial difference of the generalized metric from the metric is that the range of generalized metric includes the infinity.

In the sequel, we bring back few fundamental ideas concerning fixed point theory which will be a backbone to obtain the major results of this study. The following significant theorem was proved in [4].

Theorem 2.3. *[4]* **Fixed point alternative theorem**. *Let $(\mathcal{P}, \rho)$ be a complete generalized metric space and let $\Gamma : \mathcal{P} \longrightarrow \mathcal{P}$ be a strictly contractive mapping (that is $\rho(\Gamma(p), \Gamma(q) \leq M\rho(p, q)$, for all $p, q \in \mathcal{P}$ and a Lipschitz constant $0 < M < 1$). Then for each given element $p \in \mathcal{P}$, either $\rho\left(\Gamma^m p, \Gamma^{m+1} p\right) = \infty$ for all non-negative integers m or there exists a positive integer m_0 such that*

(i) *$\rho\left(\Gamma^m p, \Gamma^{m+1} p\right) < \infty$ for all $m \geq m_0$;*
(ii) *the sequence $\{\Gamma^m\}$ converges to a fixed point $q^\star$ of Γ;*
(iii) *$q^\star$ is the unique fixed point of Γ in the set $\Sigma = \{q \in \mathcal{P} | \rho(\Gamma^{m_0)} p, q) < \infty\}$;*
(iv) *$\rho(q, q^\star) \leq \frac{1}{1-M}\rho(q, \Gamma q)$ for all $q \in \Sigma$.*

We apply the above fixed point theorem to prove the stability of equation (2.1). To arrive at the major results, for a mapping $Q : \mathbb{A} \longrightarrow \mathbb{R}$, we designate a difference operator $D : \mathbb{A} \times \mathbb{A} \longrightarrow \mathbb{R}$

as follows:

$$DQ(p,q)=Q\left(\frac{q}{1+\frac{2q}{p}}\right)+Q\left(\frac{q}{1-\frac{2q}{p}}\right)-Q\left(\frac{q}{2+\frac{q}{p}}\right)-Q\left(\frac{q}{2-\frac{q}{p}}\right)-30Q(p)+30Q(q)$$

for all $p,q\in\mathbb{A}$.

Here, we prove the Găvruţa stability for the quartic reciprocal functional equation.

Theorem 2.4. *Suppose that a mapping $Q:\mathbb{A}\longrightarrow\mathbb{R}$ satisfies the following approximation for all $p,q\in\mathbb{A}$,*

$$|DQ(p,q)|\leq\beta\left(\frac{1}{p},\frac{1}{q}\right),\tag{2.2}$$

where $\beta:\mathbb{A}\times\mathbb{A}\longrightarrow[0,\infty)$ is a given function. Suppose a constant $0<M<1$ exists such that the mapping defined via $p\mapsto\alpha(p)=\beta\left(0,\frac{1}{p}\right)$ has the property that $\alpha\left(\frac{p}{2}\right)\leq 16M\alpha(p)$, for all $p\in\mathbb{A}$ and the mapping β satisfies

$$\lim_{m\to\infty}2^{-4m}\beta\left(\frac{2^{-m}}{p},\frac{2^{-m}}{q}\right)=0,\quad \text{for all } p,q\in\mathbb{A}.\tag{2.3}$$

Then, there exists a unique quartic reciprocal mapping $\mathcal{Q}:\mathbb{A}\longrightarrow\mathbb{R}$ such that

$$|Q(p)-\mathcal{Q}(p)|\leq\frac{1}{1-M}\alpha(p),\tag{2.4}$$

for all $p\in\mathbb{A}$.

Proof. Let us define a set B as the set of functions from $\mathbb{A}$ to $\mathbb{R}$ and introduce the generalized metric ρ on B as:

$$\rho(f,g)=\rho_\alpha(f,g)=\inf\{K>0:|f(p)-g(p)|\leq K\alpha(p),\ \text{for all } p\in\mathbb{A}\}.\tag{2.5}$$

One can easily verify that (B,ρ) is complete. Also, let $\eta:B\longrightarrow B$ be a mapping defined by

$$\eta g(p)=\frac{1}{16}g\left(\frac{p}{2}\right),\quad (p\in\mathbb{A}),$$

for all $g\in B$. Let us claim that η is strictly contractive on B. For given $f,g\in B$, let $K_{fg}\in[0,\infty]$ be an arbitrary constant with $\rho(f,g)\leq K_{fg}$. Consequently, $\rho(f,g)<K_{fg}$ and so $|f(p)-g(p)|\leq K_{fg}\alpha(p)$ for all $p\in\mathbb{A}$. This implies that

$$\left|\frac{1}{16}f\left(\frac{p}{2}\right)-\frac{1}{16}g\left(\frac{p}{2}\right)\right|\leq\frac{1}{16}K_{fg}\alpha\left(\frac{p}{2}\right),$$

for all $p\in\mathbb{A}$. Thus,

$$\left|\frac{1}{16}f\left(\frac{p}{2}\right)-\frac{1}{16}g\left(\frac{p}{2}\right)\right|\leq MK_{fg}\alpha(p),$$

for all $p\in\mathbb{A}$. Therefore, $\rho(\eta f,\eta g)\leq MK_{fg}$. Hence, we find that $\rho(\eta f,\eta g)\leq M\rho(f,g)$ for all $f,g\in B$, that is, ρ is strictly contractive mapping of B, with the Lipschitz constant M. Now, swapping (p,q) with $(0,q)$ in (2.2), we obtain

$$\left|\frac{1}{6}Q\left(\frac{p}{2}\right)-Q(p)\right|\leq\beta(0,q)=\alpha(p),$$

for all $p\in\mathbb{A}$. Therefore, from (2.5), we get $\rho(\eta Q,Q)\leq 1$. Hence, by the application of fixed point alternative Theorem 2.3, we find a function $\mathcal{Q}:\mathbb{A}\longrightarrow\mathbb{R}$ satisfying the following:

(i) $\mathcal{Q}$ is a fixed point of ρ, that is

$$\mathcal{Q}\left(\frac{p}{2}\right) = 16\mathcal{Q}(p), \tag{2.6}$$

for all $p \in \mathbb{A}$. The mapping $\mathcal{Q}$ is the unique fixed point of ρ in the set

$$\lambda = \{h \in B : \rho(Q, h) < \infty\}.$$

This implies that $\mathcal{Q}$ is the unique mapping satisfying (2.6) such that there exists $K > 0$ satisfying

$$|Q(p) - \mathcal{Q}(p)| \leq K\alpha(p)$$

for all $p \in \mathbb{A}$.

(ii) We have $\rho(\eta^m Q, \mathcal{Q}) \to 0$ as $m \to \infty$. Hence, we get

$$\lim_{m\to\infty} 2^{4m} Q\left(2^{-m} p\right) = \mathcal{Q}(p), \tag{2.7}$$

for all $p \in \mathbb{A}$.

(iii) We have $\rho(Q, \mathcal{Q}) \leq \frac{1}{1-M}\rho(Q, \eta Q)$, which implies $\rho(Q, \mathcal{Q}) \leq \frac{1}{1-M}$.

Thus, inequality (2.4) is now valid. Further, by using (2.2), (2.3) and (2.7), we obtain

$$|D\mathcal{Q}(p,q)| = \lim_{m\to\infty} 2^{4m} DQ\left(2^{-m}p, 2^{-m}q\right) \leq \lim_{m\to\infty} 2^{4m}\beta\left(\frac{2^{-m}}{p}, \frac{2^{-m}}{q}\right) = 0,$$

for all $p, q \in \mathbb{A}$. Hence, $\mathcal{Q}$ satisfies (2.1) and therefore $\mathcal{Q} : \mathbb{A} \longrightarrow \mathbb{R}$ is a quartic reciprocal mapping. Next, we show that $\mathcal{Q}$ is unique. Suppose $\mathcal{Q}' : \mathbb{A} \longrightarrow \mathbb{R}$ is another quartic reciprocal mapping satisfying (2.1) and (2.4). Due to the fact that the mapping $\mathcal{Q}'$ is a fixed point of ρ and $\rho(Q, \mathcal{Q}') < \infty$, we have $\mathcal{Q}' \in B^\star = \{h \in B | \rho(f, h) < \infty\}$. From Theorem 2.3 (iii) and since both $\mathcal{Q}$ and $\mathcal{Q}'$ are fixed points of ρ , we have $\mathcal{Q} = \mathcal{Q}'$. Therefore, we conclude that $\mathcal{Q}$ is unique. This completes the proof. □

Similar to Theorem 2.4, we have the following result. Since the proof is similar, it is omitted.

Theorem 2.5. *Suppose that a mapping $Q : \mathbb{A} \longrightarrow \mathbb{R}$ satisfies the inequality (2.1). If $0 < M < 1$ is a constant such that the mapping $p \mapsto \alpha(p) = 16\beta(0, 2q)$ has the property that $\alpha(2p) \leq \frac{1}{16}M\alpha(p)$, for all $p \in \mathbb{A}$ and a mapping $\beta : \mathbb{A} \times \mathbb{A} \longrightarrow [0, \infty)$ satisfies the condition*

$$\lim_{m\to\infty} 2^{4m}\beta\left(\frac{2^m}{p}, \frac{2^m}{q}\right) = 0, \quad \textit{for all } p, q \in \mathbb{A},$$

then there exists a unique quartic reciprocal mapping $\mathcal{Q} : \mathbb{A} \longrightarrow \mathbb{R}$ such that

$$|Q(p) - \mathcal{Q}(p)| \leq \frac{1}{1-M}\alpha(p),$$

for all $p \in \mathbb{A}$.

In the upcoming corollaries, we present the stability results of equation (2.1) under different upper bounds by applying Theorem 2.4 and Theorem 2.5.

Corollary 2.6. *Suppose that a mapping $Q : \mathbb{A} \longrightarrow \mathbb{R}$ satisfies the inequality*

$$|DQ(p,q)| \leq \delta,$$

for all $p, q \in \mathbb{A}$, where $\delta > 0$ is a constant which is independent of p, q. Then, there exists a unique quartic reciprocal mapping $\mathcal{Q} : \mathbb{A} \longrightarrow \mathbb{R}$ which satisfies (2.1) such that

$$|Q(p) - \mathcal{Q}(p)| \leq 2\delta,$$

for all $p \in \mathbb{A}$.

Proof. The desired result follows from Theorem 2.4 by considering $\beta\left(\frac{1}{p},\frac{1}{q}\right)=\delta$, for all $p,q\in\mathbb{A}$ and $M=\frac{1}{2}$. $\square$

Corollary 2.7. *Suppose that $Q:\mathbb{A}\longrightarrow\mathbb{R}$ is a mapping fulfills the approximation*

$$|DQ(p,q)|\leq\theta\left(\left|\frac{1}{p}\right|^a+\left|\frac{1}{q}\right|^a\right),$$

for all $p,q\in\mathbb{A}$. Then, there exists a unique quartic reciprocal mapping $\mathcal{Q}:\mathbb{A}\longrightarrow\mathbb{R}$ which satisfies (2.1) such that

$$|Q(p)-\mathcal{Q}(p)|\leq\begin{cases}\frac{2^{a+4}\theta}{2^{a+4}-1}\left|\frac{1}{p}\right|^a;\\ \\ \frac{\theta}{1-2^{a+4}}\left|\frac{1}{p}\right|^a,\end{cases}$$

for all $p\in\mathbb{R}$.

Proof. The desired result can be attained via Theorems 2.4 and 2.5 by choosing $\beta\left(\frac{1}{p},\frac{1}{q}\right)=\theta\left(\left|\frac{1}{p}\right|^a+\left|\frac{1}{q}\right|^a\right)$ and respectively, $M=2^{-a-4}$ and $M=2^{a+4}$. $\square$

3. Relevance of equation (1.5) to the Rayleigh scattering formula

Rayleigh's scattering formula states that the intensity of light (I) is inversely proportional to the fourth power of the wavelength (λ). That is, $I\propto\frac{1}{\lambda^4}$. If p is the wavelength of a light, then by the solution of equation (1.5), we can say $Q(p)$ is the intensity of light. Hence, the solution of equation (1.5) suggests that equation (1.5) can be related with the Rayleigh's scattering formula. The left-hand side of equation (1.5) represents the sum of the intensities of light corresponding to the wavelengths $\frac{pq}{p+2q}$ and $\frac{pq}{p-2q}$. This sum is equal to combined intensities of light corresponding to the wavelengths $\frac{pq}{2p+q}$, $\frac{pq}{2p-q}$ together with an additional contribution of 30 times the intensity of light at wavelength p and a subtraction of 30 times the intensity of light at wavelength q. Therefore, equation (1.5) expresses an intensity balance, that is, the total intensity associated with one pair of harmonically related wavelengths is conserved, up to weighted contributions from the fundamental wavelengths p and q.

3.1. Concluding remarks of obtained results with harmonic reciprocal points. Suppose p and q are two fixed points on the real line. Then, the harmonic reciprocal points are obtained by taking harmonic combinations in reciprocal coordinates, which is expressed as $x=\frac{pq}{ap\pm bq}$. The harmonic reciprocal points arise in projective geometry. Let $\frac{1}{x}=\frac{1}{p}\pm\frac{2}{q}$ or $\frac{1}{x}=\frac{2}{p}\pm\frac{1}{q}$. The arguments involved in equation (1.5) form two symmetric harmonic pairs in reciprocal space. The following Figure 1 exhibits the arguments of equation (1.5) balance each other symmetrically around p and q. Hence, the quartic reciprocal mapping preserves weighted

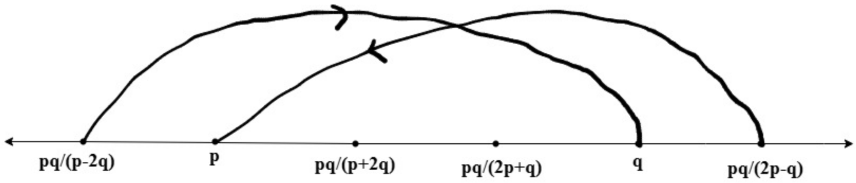

FIGURE 1. Harmonic reciprocal points $\left(\frac{pq}{p+2q},\frac{pq}{p-2q},\frac{pq}{2p+q},\frac{pq}{2p-q}\right)$

sums over harmonic reciprocal configurations. Moreover, the equation (1.5) compares the values of Q at two projectively equivalent harmonic configurations. From the stability results of equation (1.5), we observe that the function Q is almost invariant under harmonic reciprocal transformations. Thus, the stability result quantifies the rigidity of harmonic reciprocal geometry.

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References

[1] Aoki, T. (1950). On the stability of the linear transformation in Banach spaces. *J. Math. Soc. Japan*, 2, 64–66. DOI: https://doi.org/10.2969/jmsj/00210064.

[2] Bodaghi, A., & Ebrahimdoost, Y. (2016). On the stability of quadratic reciprocal functional equation in non-Archimedean fields. *Asian-European J. Math*, 9(1), 9 pp. DOI: https://doi.org/10.1142/S1793557116500029.

[3] Bodaghi, A., & Kim, S.O. (2014). Approximation on the quadratic reciprocal functional equation. *J. Func. Spac. Appl*, Art. ID. 532463, 5 pp. DOI: http://dx.doi.org/10.1155/2014/532463.

[4] Diaz, J., & Margolis, B. (1968). A fixed point theorem of the alternative for contractions on a generalized complete metric space. *Bull. Amer. Math. Soc*, 74, 305–309. DOI: https://doi.org/10.1090/S0002-9904-1968-11933-0.

[5] Găvruţa, P. (1994). A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings. *J. Math. Anal. Appl*, 184(3), 431–436. DOI: https://doi.org/10.1006/jmaa.1994.1211.

[6] Hyers, D.H. (1941). On the stability of the linear functional equation. *Proc. Natl. Acad. Sci. U.S.A*, 27, 222–224. DOI: https://doi.org/10.1073/pnas.27.4.222.

[7] Jung, S.M. (2009). A fixed point approach to the stability of the equation $f(x+y) = \frac{f(x)f(y)}{f(x)+f(y)}$. *Aust. J. Math. Anal. Appl*, 6, 1–6.

[8] Kim, S.O., Senthil Kumar. B.V., & Bodaghi, A. (2017). Approximation on the reciprocal-cubic and reciprocal-quartic functional equations in non-Archimedean fields. *Adv. Differ. Equ*, Paper No. 77. DOI: https://doi.org/10.1186/s13662-017-1128-z.

[9] Rassias, Th.M. (1978). On the stability of the linear mapping in Banach spaces. *Proc. Amer. Math. Soc*, 72, 297–300. DOI: https://doi.org/10.2307/2042795.

[10] Ravi K., & Senthil Kumar, B.V. (2010). Ulam-Găvruţa-Rassias stability of Rassias Reciprocal functional equation. *Global J. App. Math. Math. Sci*, 3, 57–79.

[11] Sadani, I. (2025). Exploring the reciprocal functional equations: approximations in diverse spaces. *Int. J. Maps Math*, 8(1), 177–191.

[12] Senthil Kumar, B.V., Al-Shaqsi, K., & Sabarinathan, S. (2021). Approximate multiplicative inverse quadratic mappings. *Adv. Math. Sci. J*, 10(1), 199–209.

[13] Senthil Kumar, B.V., Dutta, H., & Sabarinathan, S. (2020). Fuzzy approximations of a multiplicative inverse cubic functional equation. *Soft Computing*, 24, 13285–13292. DOI: https://doi.org/10.1007/s00500-020-04741-x.

[14] Ulam, S.M. (1964). Problems in Modern Mathematic, Science Editions. *John Wiley* & *Sons, Inc., New York*.